FAST AND ROBUST 3D ELECTROMAGNETIC BOREHOLE MODELING AND IMAGING ALGORITHMS. NEW JOINT INDUSTRY PROJECT PROPOSAL

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Abstract. We propose a joint industry project (JIP) in computational algorithms for the forward and inverse borehole electromagnetic problems, targeting 3D anisotropic imaging from ultra deep directional resistivity measurements. The proposal is based on new developments in PDE discretization, model order reduction, machine learning and stochastic inversion. The JIP will be a part of a broader model order reduction and physically constrained machine learning initiative at WPI, that also includes defense applications.

1. Objectives. Reliable interpretation of electromagnetic measurements in horizontal and dipping wells requires solution of forward and inverse problems for 3D Maxwell's system with *arbitrarily oriented anisotropy tensors*. In spite of recent progress in 3D electromagnetic modeling, e.g, [11], it is still at least two to three orders of magnitude slower than what would be acceptable for real-time borehole imaging on the industrial scale, e.g., deep directional resistivity and look ahead measurements in geosteering applications. Moreover, the most versatile and reliable stochastic inversion methods with uncertainty quantification require even higher speedup from the current level to become viable. This explains why the diagonally anisotropic 2.5D approximations [1, 4, 11] combined with the Gauss-Newton (GN) misfit minimization algorithm are the state-of-the-art in borehole resistivity inversion in complex formations.

Here we propose a new generation of forward modeling and inversion algorithms by reassessing our earlier finite-volume discretization algorithm for anisotropic Maxwell's system [9] and adapting some recent developments in model order reduction, machine learning and stochastic inversion. Our goal is both to dramatically reduce the computational cost and improve the reliability of full 3D anisotropic forward modeling and inversion targeting implementation on modern HPC architectures, e.g., cloud computing, GPU's, etc.

2. Fast forward modeling in complex 3D anisotropic formations. One of the most efficient and elegant mimetic finite-volume schemes for partial differential equations and systems was introduced by V.I. Lebedev in [14]. Its adaptation to the 3D anisotropic Maxwell's system obtained in [8] automatically preserves the structure of the continuum problem, such as the local conservation of electric and magnetic fluxes. Moreover, the obtained discrete Maxwell's system preserves local structure of both primary and dual systems, i.e., it does not require interpolation of electric field for computation of anisotropic Ohm's law and possess special error cancellation properties [9], that favorably differentiates it from the other mimetic discretizations used in electromagnetic computations. Enhanced by multi-scale capabilities [16] and finite-difference Gaussian quadrature rules (a.k.a. optimal grids) [13], Lebedev's grids of relatively small size are able to accurately model measurements of directional resistivity tools in *fully anisotropic, highly heterogeneous 3D formations including borehole, complex invasion, cross-bedding, fractures and coupling of bed boundaries with faults* [9, 5, 6, 7]. Thanks to its efficiency, this approach is also widely used for anisotropic elasticity problems in the seismic community, e.g., see [15].

2.1. Accelerated structure-preserving solver. The main drawback of the conventional Lebedev's scheme is that it consists of four coupled Yee grids (clusters), which leads to

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roughly 4- and 64-fold cost increase for iterative and direct solvers, respectively, e.g., see [18, 11]. Recently, we discovered an efficient approach of *splitting* the Lebedev's scheme onto four decoupled problems, that we call "decoupled structure preserving discretization" (DSPD). The decoupled problems possess the structure-preserving property of the Lebedev's discretization, hence the name. An inexpensive preprocessing and postprocessing of these four solutions yields the total solution with accuracy similar to the one of the conventional Lebedev's scheme. However, the DSPD is much faster (from one to two orders of magnitude) than the Lebedev's scheme since it employs the available parallel direct linear solvers, e.g., PARDISO and takes advantage of the "embarrassing" parallelism of the DSPD splitting. Further acceleration (about one order of magnitude) can be achieved by developing specialized semi-iterative solvers exploiting the structure of the new discretization.

The computational performance of the full 3D anisotropic DSPD solver will be comparable to the one of the diagonally anisotropic 2.5D solver [1]. However, we will also develop a 2.5D DSPD solver, that in addition of being significantly faster will also be able to handle full anisotropy. In its functionality this solver will be similar to the recent 2.5D fully anisotropic Lebedev's solver [11], but at least one order of magnitude faster.

3. Model-based inversion engines. Novel efficient forward solvers can be used stand alone or incorporated into the existing inversion frameworks. The simplest implementation would be a gradient-free fixed point approach to use the 3D solver for iterative correction of inner loop 2D or 1D "curtain" (a.k.a. stitched) inversionor to use more conventional Gauss-Newton (GN) inversion framework. However, the main remaining bottleneck of the former is slow and unreliable convergence (that can be alleviated by using Anderson's acceleration) and of the latter is handling large Jacobians, in particular for the 3D inverse anisotropic problems. Large computational cost of the GN can be alleviated via nonlinear preconditioning with 1D or 2D curtain inversion.

3.1. Novel stochastic inversion algorithm with uncertainty quantification. Stochastic inversion with uncertainty quantification is known to be superior in accuracy and reliability to deterministic methods, e. g., the GN. Since it and other gradient-based methods may easily get stuck in local minima, they have to be run from a large set of starting points in the hope of finding the global minimum. In addition, stochastic approach formulated in Bayesian framework straightforwardly incorporates priors obtained from other measurements, e.g., seismic, that becomes particularly important for the 3d inversion, which is drastically underdetermined. The conventional stochastic approach constructs the posterior probability density distribution using Markov chain Monte Carlo (MCMC) algorithms. However, the MCMC algorithms are notoriously expensive in terms of the required number of forward solves, hence their application in resistivity modeling and inversion is limited to 1D problems at the moment [19]. Novel fast DSPD solver opens new possibilities for stochastic methods in applications to 2.5D and 3D problems.

To further accelerate stochastic inversion we propose instead of MCMC to generate meaningful samples of the constitutive coefficients to be recovered by combining state-of-the-art adaptive Metropolis-Hastings algorithms and the use of multiple chains of samples [17, 12]. Our preliminary results indicate that there is a great computational advantage in using a parallel processor architecture to evaluate unscaled posteriors for proposed points in combination with a vectorized accept/reject algorithm to advance the parallel chains. In that way, *the chains are not only running in parallel, they are "learning" from one another* [3]. In preliminary work, we recovered much more accurate expected values of the coefficients as compared to the solutions obtained by a deterministic Newton method. We also reduced the number of forward solves from millions to hundreds compared to the conventional MCMC algorithm. This preliminary work pertains to the nonlinear problem consisting of reconstructing faults in seismology, that by data/model setup and computational complexity is similar to deep directional resistivity problems.

4. Data-driven inversion. In our physically constrained machine learning inversion approach the tool response is matched by a reduced-order model via a small judiciously chosen resistor-capacitor network approximating complex formation. This approach originated in electrical network synthesis and we implemented it for imaging applications such as seismic and radars, e. g., see [10, 2]. As in the most supervised learning approaches, it requires the solution of full scale forward problems only at the offline training stage. The online stage that performs actual inversion is extremely fast. Numerical experiments for 2D models imitating deep directional resistivity setups show promising, quantitatively correct inversion results. Eventually, we expect that the data-driven model reduction will greatly improve the computational efficiency for majority of the inversion scenarios, and can be used in combination with the model based inversion approaches from Section 3.

5. Summary and project plan. In addition to the authors, Dr. Leonid Knizhnerman will contribute to software development and consulting support. Some work will be done by masters and Ph.D. students at WPI in the framework of their respective masters qualifying projects and Ph.D. dissertations. We will welcome joint research and publications with member specialists.

Task	2020, Q3	2021	2022
Modeling	3D simulator with full anisotropy for major UDAR tools	fast 3D; special model features (fractures); fast 2.5D with 3D anisotropy	super-fast 2.5D and 3D
Inversion, model based		3D gradient-free deterministic	3D Gauss-Newton and stochastic
Inversion, data-driven		proof of concept 2D (2.5D geometry)	α version, 3D and 2D

TABLE 5.1
Preliminary outline of three year plan with major deliverables.

The planning start date of the consortium is Q3 of 2020. We will begin with the release of a fast fully anisotropic 3D simulator for ultra deep azimuthal resistivity (UDAR) tools of major vendors with possible addition of other directional resistivity tools by members request. We will interface our modeling and inversion software with general public domain and commercially available platforms, such as ParaView and MATLAB and provide plug-ins into standard and specific industry formats.

The consortium will comprise a mix of short- and mid-term projects. The project outlook with deliverables for a 3 year horizon is summarized in Table 5.1, assuming support of at least 5 project members.

6. Membership. For each JIP member, the yearly membership fee is set when that member joins the JIP and is determined according to the schedule in Table 6.1.

Members joining in	2020	2021	2022 and later
Yearly membership fee, USD	50,000	55,000	60,000

TABLE 6.1
Membership fee schedule.

For members joining in 2020 we expect commitment in Q1, and full payment $50,000$ USD made between May 1st and July 31st.

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